Diatonic Modes Ranked by “Brightness”

brighter

Lydian
Ionian
Mixolydian
Dorian
Aeolian
Phrygian
Locrian
darker
“Modes” (Inversions) of the Minor Triad

{0 3 7} = C minor \( _3 \)

{0 4 9} = A minor \( _4 \)

{0 5 8} = F minor \( _4 \)
TWO WAYS TO DEFINE BRIGHTNESS

1. Sum Brightness
2. Voice-Leading Brightness
Sum Brightness

Aeolian

\[
\begin{array}{cccccc}
0 & 2 & 3 & 5 & 7 & 8 & 10 \\
\end{array}
\]

= 35

Mixo.

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 5 & 7 & 9 & 10 \\
\end{array}
\]

= 37

Lydian

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 6 & 7 & 9 & 11 \\
\end{array}
\]

= 39

Phryg.

\[
\begin{array}{ccccccc}
0 & 1 & 3 & 5 & 7 & 8 & 10 \\
\end{array}
\]

= 34

Dorian

\[
\begin{array}{ccccccc}
0 & 2 & 3 & 5 & 7 & 9 & 10 \\
\end{array}
\]

= 36

Ionian

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 5 & 7 & 9 & 11 \\
\end{array}
\]

= 38

Locrian

\[
\begin{array}{cccccccc}
0 & 1 & 3 & 5 & 6 & 8 & 10 \\
\end{array}
\]

= 33
Modes of the Minor Triad

{0 4 9} = A minor 6

Sum = 13

{0 5 8} = F minor 6

Sum = 13

{0 3 7} = C minor 6

Sum = 10

brighter

darker
# Sum Brightness for Larger Scales

## Harmonic Minor

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- **A**: Locrian #4 7
- **B**: Aeolian #7
- **C**: Locrian #6
- **D**: Ionian #5
- **E**: Dorian #4
- **F**: Phrygian #3
- **G**: Lydian #2

## “Blues Scale”

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- **A**: E (31)
- **B**: F (31)
- **C**: C (31)
- **D**: A (25)
- **E**: B (25)

**brighter**

**darker**

**D (37)**

**E (31)**

**F (31)**

**C (31)**

**A (25)**

**B (25)**
Voice-Leading Brightness

In the voice leading from Locrian to Phrygian, all the motion is ascending. Therefore Phrygian is brighter.
Voice-Leading Brightness

A harmonic minor; sum = 36

A phrygian dominant; sum = 35
Voice-Leading Brightness

A harmonic minor; sum = 36

A phrygian dominant; sum = 35

The voice leading from Harmonic Minor to Phrygian Dominant lowers scale degrees 2 and 7, but it raises scale degree 3.

Contrary motion means the modes are incomparable.
Sum vs. VL Brightness for Harmonic Minor

**Sum Brightness Graph**

- **brighter**
  - G (40)
  - D (39)
  - E (37)
  - B (36) = Harmonic Minor
  - F (35) = Phrygian Dominant
  - C (34)
  - A (31)

- **darker**

**VL Brightness Graph**

- G (40)
- E (37)
- D (39)
- F (35)
- B (36)
- C (34)
- A (31)
Important Theoretical Questions

1. How does a brightness graph reflect a scale’s internal structure?

2. What is the relationship between sum brightness and voice-leading brightness?

3. How many different brightness graphs are possible?

4. What determines brightness comparisons between different set classes?
BRIGHTNESS GRAPHS & SCALE STRUCTURE

Voice Leading & Scalar Transposition
Mode Change

Scalar transposition & chromatic transposition nearly cancel out.
## Scalar Interval Matrix

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Scalar Interval Matrix

Ionian to Mixolydian

\[ t_4 = (+7,+7,+7,+7,+7,+7,+6) \]
\[ + T_{-7} = (-7,-7,-7,-7,-7,-7,-7) \]
\[ = (0,0,0,0,0,0,-1) \]

\[
\begin{array}{cccccccc}
\text{Locrian} & 0 & 1 & 3 & 5 & 6 & 8 & 10 \\
\text{Ionian} & 0 & 2 & 4 & 5 & 7 & 9 & 11 \\
\text{Dorian} & 0 & 2 & 3 & 5 & 7 & 9 & 10 \\
\text{Phrygian} & 0 & 1 & 3 & 5 & 7 & 8 & 10 \\
\text{Lydian} & 0 & 2 & 4 & 6 & 7 & 9 & 11 \\
\text{Mixolydian} & 0 & 2 & 4 & 5 & 7 & 9 & 10 \\
\text{Aeolian} & 0 & 2 & 3 & 5 & 7 & 8 & 10 \\
\end{array}
\]
### Scalar Interval Matrix

#### Locrian to Lydian

\[ t_4 = (+6,+7,+7,+7,+7,+7,+7) \]

\[ + T_{-6} = (-6,-6,-6,-6,-6,-6,-6) \]

\[ = (0,+1,+1,+1,+1,+1,+1) \]

<table>
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<th>Dorian</th>
<th>Phrygian</th>
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</table>
Scalar Interval Matrix

Harmonic Minor B to F

\[ t_4 = (+7, +6, +8, +7, +7, +7, +6) \]
\[ + T_{-7} = (-7, -7, -7, -7, -7, -7, -7) \]
\[ = (0, -1, +1, 0, 0, 0, -1) \]
SUM vs. VOICE-LEADING

Scales with a brightness “semiorder”
VL vs. Sum Brightness

Voice Leading Brightness $\Rightarrow$ Sum Brightness
VL vs. Sum Brightness

Voice Leading
Brightness

Sum
Brightness

🚫
**VL vs. Sum Brightness**

**Voice Leading Brightness**

**Sum Brightness**

*Harmonic Minor: Many VL ties although all sums are different.*

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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- A: 31
- B: 36
- C: 34
- D: 39
- E: 37
- F: 35
- G: 40
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

Difference = 1

not comparable
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

Difference = 1

not comparable
Semiordered Scales

Melodic Minor
Voice-Leading Brightness Graph:

Difference = 1

not comparable
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

\[
\begin{align*}
& \text{D (40)} \\
& \text{E (38)} \\
& \text{F (36)} \\
& \text{G (34)} \\
& \text{C (35)} \\
& \text{B (37)} \\
& \text{A (32)}
\end{align*}
\]

\text{Difference} = 1

not comparable
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

Difference = 2

comparable: E < D
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

Difference = 2

comparable: $F < E$
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

Difference = 3

comparable: C < E
Semiordered Scales

Melodic Minor

Voice-Leading Brightness Graph:

- Difference = 2

comparable: C < B
Semiordered Scales

\[ \delta : \text{The largest sum difference between incomparable modes} \]

\[ \varepsilon : \text{The smallest sum difference between comparable modes} \]
Semiordered Scales

Melodic Minor

$\delta = 1; \varepsilon = 2$
Semiordered Scales

VL Brightness for Melodic Minor & Diatonic Modes Together


Mel. Min. C (Phrygian #6) ← Mel. Min. B (Melodic Minor)

darker ← brighter
Don Giovanni Overture
Don Giovanni Overture

Melodic Minor & Diatonic

- Mel. Min. G (Locrian #2)
- Mel. Min. F (Aeolian #3)
- Mel. Min. E (Acoustic)
- Mel. Min. A (Locrian #4)
- Mel. Min. C (Phrygian #6)
- Mel. Min. B (Melodic Minor)

darker → brighter
OPTIC Spaces for Trichords
THE UPSHOT:

A scale’s structure depends only on its location in this space.

(The space partitions into regions of constant structure.)

(“Structure” includes the VL brightness graph, δ/ε, and everything they imply, e.g. specific vs. generic intervals, well-formedness, and so on.)
All Possible Trichord Brightness Graphs

Type 1a

C

B

A

Type 1b

B

C

A

Type 2

B → C

A

Type 2 (inverted)

A → B

C
Modes of (037)

{0 5 8} 13  
“second inversion”

{0 3 7} 10  
“root position”

{0 4 9} 13  
“first inversion”

Defining Inequalities

\[ x_1 - x_0 < x_2 - x_1 \]

pitches 0 3 7  (12)

intervals 3 < 4
PT Space for Trichords

Inequality #1
\[ x_1 - x_0 < x_2 - x_1 \]

Inequality #2
\[ x_1 - x_0 < 12 + x_0 - x_2 \]

Inequality #3
\[ x_2 - x_1 < 12 + x_0 - x_2 \]
PT Space for Trichords

Graph Type 1a
- C
- B
- A

Graph Type 1b
- B
- C
- A

Graph Type 2
- B
- C
- A

Graph Type 2 (inverted)
- A
- B
- C
Graph of Trichord Space
PT Space for Trichords
PT Space for Trichords
PT Space for Trichords

Color indicates $\delta/\varepsilon$.
Red shows $\delta/\varepsilon = 0$.
Purple shows $\delta/\varepsilon = 1$. 
PT Space for Trichords
mapped onto $S^1$ (a circle)
PT Space for Tetrachords
mapped onto $S^2$ (a sphere)

Color indicates $\delta/\varepsilon$. Red shows $\delta/\varepsilon = 0$. Blue shows $\delta/\varepsilon = 1$. As $\delta/\varepsilon$ grows without bound, the color approaches violet.
Modes of Harmonic Minor \((\delta=2; \varepsilon=3)\)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Chords</th>
<th>Notes</th>
<th>Mode Type</th>
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<td>0 2 3 5 7 8 11</td>
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<td>G</td>
<td>0 3 4 6 7 9 11</td>
<td>0 3 4 6 7 9 11</td>
<td>Lydian 2</td>
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Mode (sum)


darker  brighter
Modes of Harmonic Minor \((\delta=2; \varepsilon=3)\)

Mode (sum)

A (31)  \(\rightarrow\) C (34)  \(\rightarrow\) F (35)  \(\rightarrow\) B (36)  \(\rightarrow\) E (37)  \(\rightarrow\) D (39)  \(\rightarrow\) G (40)

darker \(\rightarrow\) brighter

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<td>G</td>
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<td>6</td>
<td>7</td>
<td>9</td>
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</tbody>
</table>

- A: Locrian \(b4\); \#7
- B: Aeolian \#7
- C: Locrian \#6
- D: Ionian \#5
- E: Dorian \#4
- F: Phrygian \#3
- G: Lydian \#2


